JOURNAL OF AIRCRAFT
Vol. 30, No. 1, Jan.-Feb. 1993

Sound Generation by Rotor-Vortex Interaction in Low Mach Number Flow

F. Obermeier* and K.-Q. Zhu†
Max-Planck-Institut für Strömungsforschung, D-3400 Göttingen, Germany

The primary components of helicopter noise consist of periodic, impulsive sound waves generated by vortex filaments which—after separation from the tip of rotor blades—interact with the following rotor blades (BVI, blade-vortex interaction noise) or are chopped by the tail rotor. During the unsteady interaction of rotor blades with vortex filaments, additional vortex sheets separate from the trailing edge of the blades (Kutta condition). This article deals mainly with two problems concerning the prediction of aerodynamic sound generation: 1) the contributions to the BVI sound field due to unsteady separation of vortex sheets at sharp trailing edges; and 2) the effects on the predicted sound field due to simplifications concerning the description of the motion of the vorticity, e.g., due to a frozen-field assumption. The general considerations are illustrated by numerical results obtained for simple flow geometries. They show that especially the sound radiated in mean flow direction depends sensitively on the approximation introduced, and that neglecting the Kutta condition and/or the frozen-field assumption will yield incorrect results.

Nomenclature

a	= radius (z-plane)
a_0	= speed-of-sound
b_1	= dipole moment
$2c$	= coordinate of the trailing edge of the airfoil in die ζ-plane
G	= Green's function
$\tilde{G}$	= Green's vector function
g	= vector function
L	= actual chord length of the airfoil
M	= U_0/a_0 Mach number
p	= pressure
T, $\tilde{T}$	= time coordinate
U_H	= velocity at the trailing edge of the airfoil
U_0	= mean velocity
u	= velocity
w	= vorticity
X, $\tilde{X}$	= space coordinates
x, y	= space coordinates
Y_i, $i = 1, 2, 3$	= normalized flow potentials
z	= complex coordinate (transformed plane)
z_M	= center of transformed Joukowski profile
Γ	= circulation
$\gamma = \Gamma/LU_0$	= nondimensional circulation
δ	= Dirac function
ζ	= complex coordinates (physical plane)
ρ_0	= ambient density
τ	= time
ϕ	= flow potential
ψ_i	= two-dimensional stream function

Subscript

k,m	= summation indices (number of vortices)

Presented as Paper 90-3974 at the AIAA 13th Aeroacoustic Conference, Tallahassee, FL, Oct. 22–24, 1990; revision received Feb. 3, 1992; accepted for publication Feb. 7, 1992.

*Professor.
†Professor; permanent address: Department of Modern Mechanics, The University of Science & Technology of China, Hefei 230026, People's Republic of China.

I. Introduction

AERODYNAMIC noise generation due to the interaction of unsteady flow with rigid bodies becomes increasingly important as one of the major sources of flow noise. Well-known examples include free jets, which may interact with the landing flaps of an aircraft, or tip vortices separated from rotor blades of a helicopter which either interact with the following blades or are chopped by the tail rotor. During the unsteady vortex-rotor interaction (which affects the lifting force acting on the rotor blades), vortex sheets separate from the trailing edges of the rotor in such a way that the velocity field and the pressure distribution behave regularly at the trailing edges. The flowfield fulfills a Kutta condition (K.C.) at the trailing edge.

The general geometry of such a flowfield is very complex and highly three-dimensional and there are still many unresolved questions left with respect to aerodynamic sound generation. This article will emphasize the basic mechanisms of sound generation as far as they are connected with blade-vortex interactions (BVI), especially the effects on the sound field due to an imposition of the Kutta condition and due to the simplifying frozen-field assumptions for the motion of the vorticity are discussed, but this article will not deal with technical aspects, they were previously outlined in a survey paper by Leverton.[1]

Despite the fact that the basic equations of fluid motion can be replaced by an integral equation (which was independently derived by Möhring et al.[2] and by Ffowcs Williams and Hawkings[3]), an adequate analytical description of these problems seems to still be beyond the present possibilities. An evaluation of that equation for the flow geometry of rotating blades of a helicopter in flight seems promising only in terms of numerical calculations which require detailed knowledge of rotor aerodynamic loads and flowfield information to predict the noise.

The incoming vorticity has often been modeled in the literature by shear waves which pass the rotor blade with constant mean velocity. Mathematically, this approximation implies that the basic equations of motion are linearized with respect to the unsteady flow contribution by the incoming vorticity. This scheme then allows for the derivation of a linear-convected wave equation for the acoustic pressure field where the convective terms depend only on the steady mean flow around the airfoil.[4] To elaborate on the effect and shortcomings of this specific simplification and to understand the basic mechanisms of aerodynamic sound generation, we will concentrate on simple flow geometries.

For that purpose all typical flow velocities are assumed to be small compared with the speed-of-sound, and all typical flow lengths small compared with the wave length of the generated sound field. This kind of flowfield is called "acoustically compact," furthermore, the part of the flowfield close to the airfoil is called the inner-flow region. Mathematically, this inner-flow region can be described by the incompressible approximation of the equations of fluid dynamics.

The far-field or the outer-flow region, where acoustical properties of the flow are dominant, is described by a linear convected wave equation. For acoustically compact flows the corresponding solution of the wave equation can be determined completely from the knowledge of the far-field expansion of the incompressible approximation of the inner-flow region, as has been shown before by means of the method of matched asymptotic expansions.[2,5,6]

In the case of blade-vortex interaction, a dipole-like sound field is expected during the time vorticity passes the airfoil. Furthermore, in low Mach number flow that dipole-like source distribution appears to be fixed at the center of the airfoil for observation points in the sound-field region.

In the following, the investigation is confined to the inner solutions of the pressure fluctuations and of the vortex motion only; the corresponding sound field will not be discussed in detail. Flow noise caused by two-dimensional line vortices moving in the mean flow of a two-dimensional airfoil where the axes of the vortex and the airfoil are perpendicular or parallel to each other is investigated in detail. In the latter case, the question of how the fulfillment of a K.C. at the sharp trailing edge of the airfoil (which implies the separation of a vortex sheet) might affect the motion of the vortices and the generated sound field is addressed.

This question has been investigated both experimentally and theoretically before (e.g., Refs. 7–9). Furthermore, the corresponding flow along a semi-infinite plate instead of a flow around an airfoil has been discussed.[10–12] Howe[12] claims that in the case of a K.C. fulfilled at the trailing edge of the plate, the generated sound field turns out to be identical with the one produced by corresponding vortices moving along an infinite wall; i.e., he claims that the typical monopole-dipole-like sound field with its cardioid-like directivity vanishes if the K.C. is fulfilled. This statement, however, is not completely correct. It is correct only if one additionally assumes that all vortices (i.e., the primary vortices as well as the vortex sheet separating from the trailing edge of the plate) move with the same constant mean flow velocity parallel to the plate. But this is an additional requirement that is known as the frozen field assumption. It cannot be deduced from the K.C. and in reality it is not fulfilled. In a recent article Howe[13] discussed the interaction of a single vortex with a two-dimensional airfoil in low Mach number flow where the axes of the vortex and the airfoil are no longer parallel. Here it is again assumed that the incoming vortices as well as the vorticity shed from the sharp trailing edge are convected by the mean flow. The limits of such an assumption will be discussed in more detail.

In the first section a vortex theory of aerodynamic sound generation[14,15] was developed and is shortly summarized. This theory is then applied to the blade-vortex interaction in Sec. III. Here, the sound generation by vortices whose axes are parallel to a rotor blade is discussed in detail, including the effects due to the fulfillment of a K.C. at the sharp trailing edge of the rotor blade and to the frozen-field assumption, respectively. Finally, numerical results are presented in Sec. V.

II. Theory

As has been shown by Möhring[14] and Obermeier,[15] the incompressible approximation of the pressure fluctuations in the inner-flow region depends linearly on the vorticity variations

$$p + \frac{1}{2}|u|^2 = \int \tilde{G}\cdot\frac{\partial}{\partial\tau}\,\mathrm{curl}\,u\,\mathrm{d}V \tag{1}$$

with

$$\mathrm{grad}\,G = \mathrm{curl}\,\tilde{G} + g$$

Here all flow quantities have been rendered dimensionless in terms of the chord length L of the airfoil, U_0 at infinity, and ρ_0. G is the ordinary Green's function for the Laplace equation which accounts for the boundary condition $\partial G/\partial n = 0$ at the surface of the airfoil. g is a function which has to be chosen in such a way that

$$\int (u \times \mathrm{curl}\,u)\cdot g\,\mathrm{d}V = 0$$

Due to Howe[13] G may be approximated for low Mach number flow very elegantly by

$$G = \frac{1}{4\pi}|x - Y(y)|^{-1}$$

where the components Y_i, $i = 1, 2, 3$, describe three normalized potentials of stationary flows in x_i direction, respectively, around the body in question. The corresponding $\tilde{G}$ can be determined explicitly as well.[16]

This general theory becomes particularly simple for three-dimensional vortex flows around two-dimensional cylindrical bodies which extend (for instance in x_3 direction) to infinity. We get

$$\lim_{|x|\to\infty} \tilde{G} = [-x_3y_2, x_3y_1, x_1\psi_1(y) + x_2\psi_2(y)]\,\frac{1}{4\pi|x|^3} \tag{2}$$

where $\psi_j(y)$ are two-dimensional stream functions of a potential flow around the cylindrical body in x_j direction, $j = 1, 2$.

Introducing Eq. (2) into the general representation for the pressure field one obtains

$$p = \frac{x_i}{4\pi|x|^3}\int \psi_i(y_1, y_2)\,\frac{\partial w_3(y)}{\partial\tau}\,\mathrm{d}^3y \tag{3}$$

where $w_3 = (\mathrm{curl}\,u)_3$. All other terms cancel. Consequently, only the component of the vorticity, which is parallel to the axis of the cylindrical body, contributes to the sound field. Furthermore, the pressure fluctuations depend exclusively on the rate at which the x_3-component of the vorticity crosses the streamlines $\psi_i = $ const. This outcome allows two already important conclusions. First, there would be no radiation into the mean flow direction—where the actual streamlines of the mean flow coincide with $\psi_1 = $ const—if the vorticity is accepted to be convected along $\psi_1 = $ const. Second, if it is assumed that the vorticity passes the airfoil with constant velocity on straight trajectories (frozen field) it would cross the streamlines ψ_1 at a rate different from the actual one and, consequently, would yield a wrong sound field. This outcome is valid regardless of whether the vorticity is given by single incoming vortices or by a turbulent gust often described mathematically in terms of shear waves.

III. Blade-Vortex Interaction

The general theory to helicopter blade-vortex interaction noise is now applied. Here we will concentrate on two cases:

1) In Fig. 1 a rotor cuts normal through a vortex. In this case only a small portion of the blade interacts with the vortex, the overall effects on the blade (and, consequently, on the sound generation) should be small.

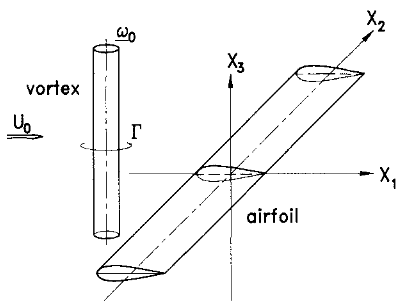


Fig. 1 Vortex chopping; the axis of the approaching vortex is perpendicular to the axis of the airfoil.

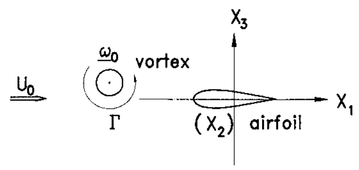


Fig. 2 Two-dimensional blade-vortex interaction; the axes of the vortex and airfoil are parallel.

2) In Fig. 2 a rotor blade moves nearly parallel to the axis of the vortex and, therefore, interacts in its full length with the vortex. It seems reasonable that this case is the more important one with respect to aerodynamic sound generation.

A. Vortex Chopping

The first problem was only recently discussed in detail by Howe[13]; this investigation was based on Powell's approach,[17] which was extended in 1975.[12] In fact, the following vector product is used

$$L = u \times \text{curl } u$$

as a source term and, in addition, an ordinary Green's function tailored to the flow geometry in question.

Based on the general results [Eq. (3)], two different mechanisms of sound generation can be identified:

1) If the incoming line vortex has an axial (x_2) velocity component, which depends on the radial direction, then its vorticity has an azimuthal component and, consequently, a component in the x_3 direction which directly causes pressure fluctuations. Furthermore, the vortex will also be deformed due to the interaction with the airfoil which leads to additional contributions to the x_3 component of the vorticity and, therefore, to the radiated sound field.

2) The component of vorticity of the approaching vortex (which originally may point in the x_2 direction), is deformed due to the thickness of the airfoil and by the velocity induced by the primary deformation of the vortex line. These two effects also lead to a vorticity component in x_3 direction and, consequently, give rise to pressure fluctuations.

B. Two-Dimensional Blade-Vortex Interaction

The case where the axis of an incoming vortex is parallel to the airfoil (Fig. 2) can be handled in a similar way by our general theory. Here, it is important to emphasize that the sound radiation normal to the mean flow direction depends linearly on the vorticity of the incoming vortex while the radiation in flow direction increases quadratically with the vortex strength. Therefore, in many applications, the latter one is an order of magnitude smaller than the first one. Finally the fulfillment of the Kutta condition at the trailing edge is of some importance as well.

As mentioned before, the interaction of the incoming vortex with the airfoil leads to the vortex sheet separating from the sharp trailing edge of the airfoil in order to avoid singularities in the pressure and velocity distribution. This unsteady vortex sheet is modeled by a series of line vortices. Such a simplification is justified as long as one is only interested in the flow close to the airfoil where instability effects of the vortex sheet are still of secondary importance.

It is assumed that an incoming vortex with the dimensionless circulation γ_0, which moves in a superimposed mean flow along the trajectory $X_0(\tau) = [X_{10}(\tau), X_{20}(\tau)]$ around the airfoil, has been generated long before we start our actual calculation. Hence, it seems reasonable to choose the circulation γ of an undetermined steady flow around the airfoil, which formally is due to a vortex fixed at the center of the airfoil, in such a way that the K.C. at the trailing edge is fulfilled for $\tau \to -\infty$. In the case of an airfoil at zero angle of attack this assumption implies that $\gamma = \gamma_0$. Next, it is assumed that at time $\tau = 0$, when the actual calculation is started, the generated vortex sheet can be modeled by a single vortex γ_1 close to the trailing edge in $X_1 = (2c + \frac{1}{2}U_H\Delta\tau, 0)$. Here ($X_1 = 2c$, $X_2 = 0$) is the location of the trailing edge, $U_H(\tau)$ is the regular velocity in the mean flow direction at the trailing edge, and $\Delta\tau$ is the time step of the calculations. Here, one could discuss whether it may be more reasonable to replace the factor $\frac{1}{2}$ by $\frac{1}{4}$ or any other similar value, however, differences in the results obtained for the far-field pressure distribution are small.

For each following time step $\Delta\tau$ an additional vortex with the circulation γ_n is introduced into the flow at $X_n = (2c + \frac{1}{2}U_H\Delta\tau, 0)$, $n = 2, 3, \ldots$, whereby γ_n is determined in such a way that the total flow, consisting of $n + 2$ vortices and the mean flow, fulfills the K.C. at the sharp trailing edge of the airfoil. Consequently, after each time step the number of vortices increases by one. Furthermore, after a vortex is introduced into the flow, it moves only in accordance with the governing flow equations which will be given below.

To simplify the analysis, a symmetric Joukowski airfoil has been used. According to the classical wing theory, such an airfoil in the complex ζ-plane may be obtained by conformal mapping of the displaced circle k: ($|z - z_M| < a$) in the complex z-plane

$$\zeta = z + \frac{c^2}{z} \qquad \text{with} \qquad \zeta = X_1 + iX_2$$

Here $c = a + z_M$ is a transformation coefficient, a is the radius of the circle, and z_M its center. As the dimensionless chord length of the airfoil is one a is given by

$$a = 0.125 + \tfrac{1}{2}\sqrt{0.00625 - z_M}$$

Now, due to the present modeling of the flow and of the vortex sheet, the vorticity distribution is given by

$$\text{curl } u = \left\{0, 0, \sum_k \gamma_k \delta[\zeta - \zeta_k(\tau)]\right\} \tag{4}$$

Therefore, there exists a flow potential

$$\phi = \left[(z - z_M) + \frac{a^2}{(z - z_M)}\right] + \frac{\gamma_0}{2\pi i}\log(z - z_M) + \frac{1}{2\pi i}\sum_{k=0}^{k(\tau)} \gamma_k \log \frac{(z_k - z)(\bar{z}_k - z_M)}{a^2 - (\bar{z}_k - \bar{z}_M)(z - z_M)} \tag{5}$$

with

$$\zeta = z + \frac{c^2}{z}$$

z_k are the positions of vortices in the z-plane, the bar indicates conjugate complex values. The pressure field is given by

$$p = -\frac{\partial}{\partial\tau}\text{Re}(\phi) - \frac{1}{2}|\text{grad Re}(\phi)|^2 \tag{6}$$

Introducing Eq. (5) into Eq. (6) one obtains for the far-field approximation

$$\lim_{|\zeta|\to\infty} p = 1 + 2\,\mathrm{Re}\left(\frac{b_1}{\zeta}\right) + \frac{\gamma_0}{\pi}\,\mathrm{Im}\left(\frac{1}{\zeta}\right) + 0\left(\frac{1}{|\zeta|^2}\right) \tag{7a}$$

with

$$b_1 = \frac{1}{2\pi i}\frac{\partial}{\partial\tau}\sum_{k=0}^{k(\tau)}\gamma_k\left(\frac{a^2}{\bar{z}_k - \bar{z}_M} - z_k\right) \tag{7b}$$

The coefficient b_1 still depends on the trajectories of the vortices and, consequently, is time dependent. The trajectories themselves are given by Hamiltonian equations[18] which may also be derived from

$$\frac{\mathrm{d}\bar{\zeta}k}{\mathrm{d}\tau} = \lim_{\zeta\to\zeta_k}\left\{\frac{\mathrm{d}}{\mathrm{d}\zeta}\left[\phi - \frac{\gamma_k}{2\pi i}\log(\zeta - \zeta_k)\right]\right\} \tag{8}$$

where the complex flow potential is given by Eq. (5). ζ_k are the positions of the vortices in the ζ-plane.

Equation (8) simply means that each vortex moves in the field due to the velocity induced by all other vortices and in the field of the superimposed mean flow. For the actual calculations it turns out to be appropriate to solve these equations not in the ζ-plane but in the z-plane, where the trajectories Eq. (8) are described by

$$\frac{\mathrm{d}\bar{z}_k}{\mathrm{d}\tau} = \frac{1}{\left|\dfrac{\mathrm{d}A(z_k)}{\mathrm{d}z_k}\right|^2}\left\{\lim_{z\to z_k}\frac{\mathrm{d}}{\mathrm{d}z}\left[\phi - \frac{\gamma_k}{2\pi i}\log(z - z_k)\right] + \frac{\gamma_k}{4\pi i}\frac{\left[\dfrac{\mathrm{d}^2A(z_k)}{\mathrm{d}z_k^2}\right]}{\left[\dfrac{\mathrm{d}A(z_k)}{\mathrm{d}z_k}\right]}\right\} \tag{9}$$

with

$$A(z) = z + (c^2/z)$$

For further details we refer to von Friedrich-Schroeter.[20]

In the case of a flow along a semi-infinite plate the general theory still can be applied as long as the vortices are close to the trailing edge of the plate even though the plate itself is not an acoustically compact body. The flow around the trailing edge of a semi-infinite plate may also be considered as a rough model describing the interaction of acoustically noncompact vorticity with the trailing edge of a thin airfoil of finite chord length.

When the theory is applied to a semi-infinite plate, the general relations given above can be simplified considerably

$$\lim_{|\zeta|\to\infty}\phi = \zeta + \frac{1}{2\pi i}\frac{\bar{\zeta}^{1/2}}{|\zeta|}\sum_{k=0}^{k(\tau)}\gamma_k(\zeta_k^{1/2} + \bar{\zeta}_k^{1/2}) + 0\left(\frac{1}{|\zeta|}\right) \tag{10a}$$

$$\lim_{|\zeta|\to\infty} p = -\frac{1}{4\pi i}\frac{\bar{\zeta}^{1/2}}{|\zeta|}\sum_{k=0}^{k(\tau)}\gamma_k\frac{\bar{\zeta}_k^{1/2}\dfrac{\mathrm{d}\zeta_k}{\mathrm{d}\tau} + \zeta_k^{1/2}\dfrac{\mathrm{d}\bar{\zeta}_k}{\mathrm{d}\tau}}{|\zeta_k|} + 0\left(\frac{1}{|\zeta|}\right) \tag{10b}$$

The K.C. requires

$$\sum_{k=0}^{k(\tau)}\gamma_k\frac{\zeta_k^{1/2} + \bar{\zeta}_k^{1/2}}{|\zeta_k|} = 0 \tag{11}$$

Consequently, the first term in Eq. (10b), which decays like $0(\zeta^{-1/2})$ for $|\zeta|\to\infty$, vanishes for finite ζ only if in addition the following approximation is valid

$$\frac{\mathrm{d}\zeta_k}{\mathrm{d}\tau} = 1 \quad\text{and}\quad \frac{\mathrm{d}\bar{\zeta}_k}{\mathrm{d}\tau} = 1 \tag{12}$$

Mathematically, this is the frozen-field assumption. The equations which describe the trajectories of the vortices more realistically are, however

$$\frac{\mathrm{d}\bar{\zeta}_k}{\mathrm{d}\tau} = +1 - \frac{i}{8\pi}\frac{\gamma_k}{\zeta_k} - \frac{i}{4\pi}\sum_{m\neq k}^{k(\tau)}\frac{\gamma_m}{\zeta_k^{1/2}(\zeta_k^{1/2} - \zeta_m^{1/2})} + \frac{i}{4\pi}\sum_{m=0}^{k(\tau)}\frac{\gamma_m}{\zeta_k^{1/2}(\zeta_k^{1/2} + \bar{\zeta}_m^{1/2})} \tag{13}$$

They do not lead to Eq. (12) when Eq. (11) is fulfilled. Therefore, neither the requirement of the Kutta condition nor the frozen-field assumption alone yields vanishing pressure fluctuations.

IV. Sound Field

As has been mentioned before, the sound field (i.e., the outer flow region) is described by a homogeneous, convected wave equation, i.e., for the flow potential ϕ^0

$$\left(\frac{\partial}{\partial\tau} + M\frac{\partial}{\partial x_1^0}\right)^2\phi^0 - \Delta\phi^0 = 0 \tag{14}$$

where the superscript 0 refers to the outer flow domain.

Here the outer coordinates are obtained from the inner ones by $x^0 = U_0/a_0\,X$; a_0 is the speed-of-sound. The solution of Eq. (14) has to fulfill Sommerfeld's radiation conditions; furthermore, free constants of the solution can uniquely be determined from the knowledge of the far-field approximation of the inner solution by means of the method of matched asymptotic expansions. For that purpose Eq. (14) is reduced to an ordinary wave equation

$$\frac{\partial^2\phi^0}{\partial\tilde{T}^2} - \tilde{\Delta}\phi^0 = 0 \tag{15}$$

by a Galilean and Lorentz transform carried out one after another

$$\hat{X}_1 = x_1^0 - M\tau,\quad \hat{X}_2 = x_2^0,\quad T = \tau$$

$$\tilde{X}_1 = \frac{\hat{X}_1 + MT}{\sqrt{1 - M^2}},\quad \tilde{X}_2 = \hat{X}_2,\quad \tilde{T} = \frac{T + M\hat{X}_1}{\sqrt{1 - M^2}}$$

$$\tilde{\Delta} = \frac{\partial^2}{\partial\tilde{X}_1^2} + \frac{\partial^2}{\partial\tilde{X}_2^2}$$

Here, the source location remains unaffected as long as terms of the order $0(M^2)$ are negligible. The final solution for the far-field approximation of the sound field which fits all conditions imposed is given by

$$\lim_{|\tilde{X}|\to\infty} p = \frac{1}{4\pi}\frac{1}{\sqrt{2|\tilde{X}|}}\left[1 + M\cos\left(\tan\frac{\tilde{X}_2}{\tilde{X}_1}\right)\right]^{-1} \cdot\frac{\partial^2}{\partial\tilde{T}^2}\int_{-\infty}^{\tilde{T}^r}\sum_{k=0}^{k(\tau)}\gamma_k\frac{\tilde{X}_1\psi_1(\zeta_k) + \tilde{X}_2\psi_2(\zeta_k)}{|\tilde{X}|\sqrt{\tilde{T}^r - \tilde{T}'}}\,\mathrm{d}\tilde{T}' \tag{16}$$

where $\tilde{T}^r = \tilde{T} - |\tilde{X}|$.

Here we have accounted for the fact that in two dimensions the far-field approximation of the ordinary Green's function is[20]

$$\lim_{|\tilde{X}|\to\infty} G = \frac{1}{2\pi}\sqrt{\frac{1}{2|\tilde{X}|}}\frac{1}{\sqrt{\tilde{T} - \tilde{T}' - |\tilde{X}| + \dfrac{\tilde{X}\cdot\tilde{Y}}{|\tilde{X}|}}}$$

V. Results

In this section numerical results for the far-field approximation of the inner-flow region are presented.

A. Joukowski Profile

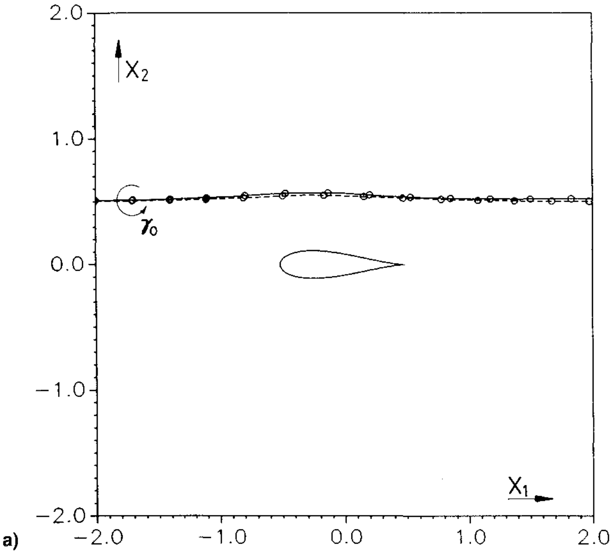
The Joukowski profile, whose interaction with an incoming vortex has been investigated, is characterized by $z_M = -0.05$ which yields for the conformal mapping circle radius $a = 0.29$. The circulation of the incoming vortex is assumed $\gamma_0 = 1$. The primary vortices start at $\tau = 0$ either in $\zeta_0 = -5 + 0.5i$ or in $\zeta_0 = -5 + 0.25i$, respectively, which in both cases means approximately five chord lengths ahead of the profile.

Figure 3 shows trajectories of the primary vortex in a flowfield which fulfills the K.C. (solid line) and which does not fulfill this condition (dashed line). These trajectories are calculated from Eqs. (5), (8), and (9). We realize that in comparison with the trajectories without the K.C., the trajectories with the K.C. turn away from the airfoil. This result agrees qualitatively well with experimental data obtained by Meier and Timm.[21] Their data are reproduced in Fig. 4.

Figure 5 shows the dipole components [Eq. (7b)] of the pressure distribution for different starting points ζ_0 of the vortex. The vortex passes the leading edge approximately at time $\tau = 4.4$, the trailing edge at $\tau = 5.4$. The main results are:

1) For the dipole component in flow direction, the deviations between the values with and without the K.C. are smaller than between the components normal to the flow direction.

2) The peaks of the fluctuations become more pronounced when the incoming vortex comes closer to the airfoil. This



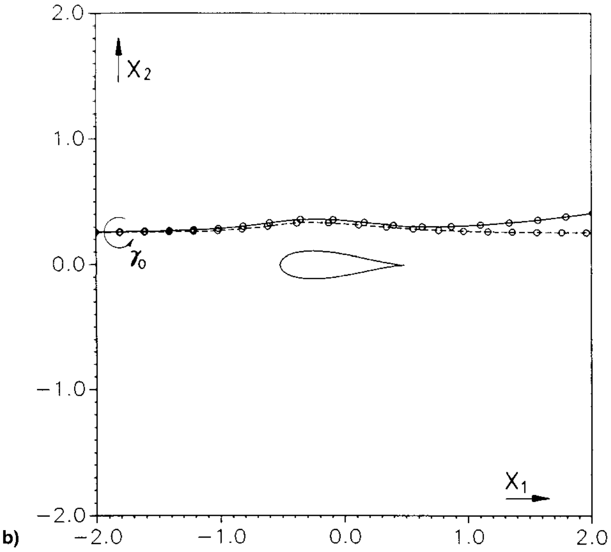


Fig. 3 Trajectories of the primary vortex with $\gamma_0 = 1$ in flows with the Kutta condition fulfilled (solid line) and without the Kutta condition fulfilled (dashed line): a) $\zeta_0 = -5 + 0.5i$, and b) $\zeta_0 = -5 + 0.25i$.

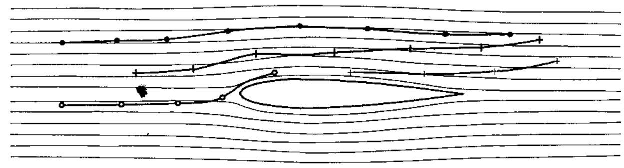


Fig. 4 Trajectories of single vortices which pass an airfoil.[22] Circulation $\gamma_0 = 0.65$, Mach number $M = 0.4$, Reynolds number $Re = 3.7 \cdot 10^5$, ——— trajectories (from interferograms), ——— streamlines of the steady mean flow (calculated).

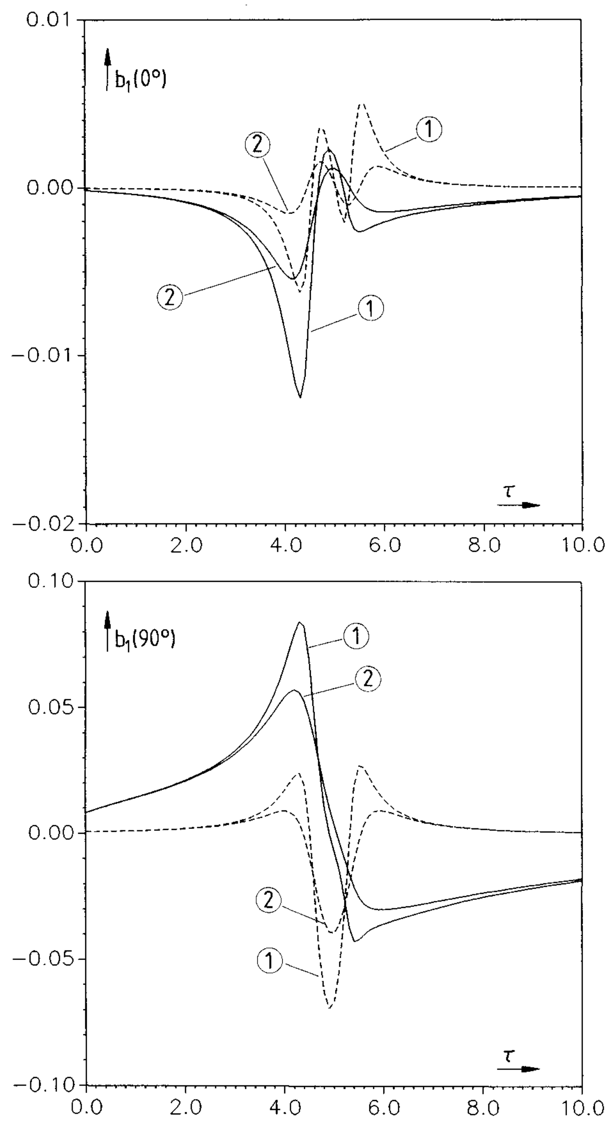


Fig. 5 Dipole coefficient b_1 [Eq. (7)] for the mean flow direction ($\phi = 0$ deg) and the normal direction ($\phi = 90$ deg) as a function of time τ. $\gamma_0 = 1$; starting point at $\tau = 0$: ① $\zeta_0 = -5 + 0.25i$; and ② $\zeta_0 = -5 + 0.5i$. ——— No Kutta condition, ——— with Kutta condition.

result has been expected as the self-induced motion of the vortex (interacting with its mirror vortex inside the circle) becomes especially large when the vortex advances the airfoil. Furthermore (not shown here), the dipole moments increase with increasing circulation as well.

In Fig. 6 results are compared where 1) the vortices move along trajectories obtained from Eqs. (5), (6), and (9); 2) the vortices move along the streamlines ψ_1 = const of the steady mean flow (simply convected); and 3) the vortices move with the constant nondimensional velocity one [Eq. (12), frozen field]. While the differences in the dipole coefficients in X_2-direction are small, the X_1-components depend strongly on the chosen trajectories of the vortices. As the frozen-field assumption has a resemblance with a strong force in the flow, which keeps the vorticity along pathes X_2 = const, the dipole moment b_1 (0 deg) in case 3 is larger than the one in the real case 1. In case 2, where the vorticity is convected along streamlines ψ_1 = const, one gets b_1 (0 deg) = 0 in accordance with the general theory [Eq. (3)].

B. Semi-Infinite Plate

The corresponding results when the incoming vortex passes the trailing edge of a semi-infinite plate are shown in Fig. 7.

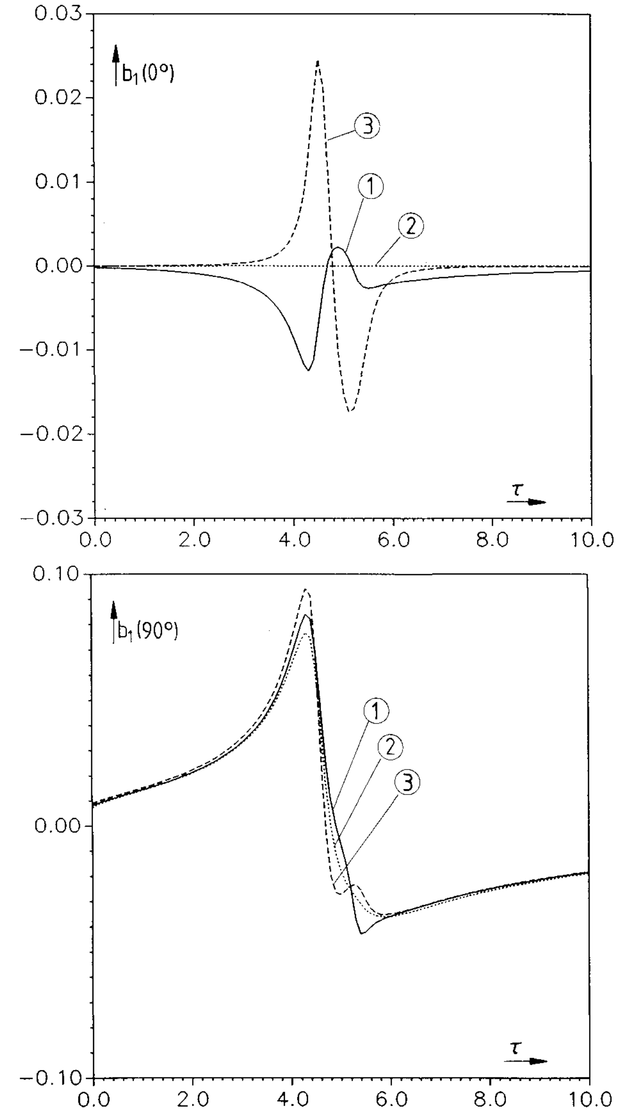


Fig. 6 Dipole coefficient b_1 for the mean flow direction (ϕ = 0 deg) and the normal direction (ϕ = 90 deg) as a function of time τ; Kutta condition fulfilled; γ_0 = 1; ζ_0 = −5 + 0.25i: ① Vortex γ_0 moves along the correct trajectory; ② vortex γ_0 moves along a streamline of the steady mean flow; and ③ vortex γ_0 moves with the constant velocity $U_0/\Gamma L$ = 1 parallel to the mean flow (frozen field).

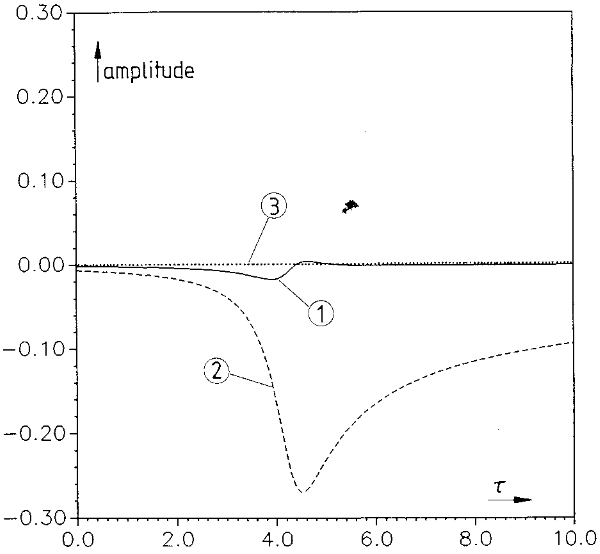


Fig. 7 Semi-infinite plate. Strength of the cardioid-like pressure distribution (normalized with $2\pi i\zeta^{1/2}$) as a function of time τ: ① Vortex γ_0 moves along the correct trajectory, Kutta condition fulfilled; ② vortex γ_0 moves along the correct trajectory, no Kutta condition fulfilled; and ③ vortices moves along the streamlines of the parallel mean flow, Kutta condition fulfilled (frozen field). The vortex γ_0 passes the leading edge at approximately τ = 4.4.

Here the pressure distribution has been normalized by $(-4\pi i\zeta^{1/2})$. These results validate the theoretical prediction; the pressure fluctuation with its cardioid-like directivity distribution obtained from Eq. (10b) vanishes only if in addition to the Kutta condition (Eq. 11) the frozen-field assumption (Eq. 12) is fulfilled. But the results also show that sound radiation is already considerably reduced when only the Kutta condition is enforced.

VI. Conclusion

Based on a general theory of low Mach number vortex noise, theoretical results on sound generation due to the interaction of line vortices with a two-dimensional airfoil and a semi-infinite plate are presented. It has been shown that the radiated sound field, especially the one calculated for the mean flow direction, depends very much on whether a Kutta condition at the trailing edge of the airfoil or of the plate is fulfilled or not. In addition, the sound field was found to be sensitive to the correct description of the trajectories of the incoming vortex and of the vortex sheet shed from the trailing edge. The last result implies that in more complex flows, where it has often been assumed in the literature that the vorticity passes the airfoil along streamlines of the mean flow or along straight lines (frozen field), the sound field has not been calculated correctly.

References

[1]Leverton, J. W., "Twenty-Five Years of Rotorcraft Aeroacoustics: Historical Prospective and Important Issues," *Journal of Sound and Vibration*, Vol. 133, 1989, pp. 261–287.

[2]Möhring, W. F., Müller, E.-A., and Obermeier, F. F., "Schallerzeugung durch instationäre Strömung als singuläres Störungsproblem," *Acustica*, Vol. 21, 1969, pp. 184–188.

[3]Ffowcs Williams, J. E., and Hawkings, D. C., "Sound Generation by Turbulence and Surfaces in Arbitrary Motion," *Philosophical Transactions of the Royal Society of London, Series A*, Vol. 264, 1969, pp. 321–342.

[4]Goldstein, M. E., *Aeroacoustics*, McGraw-Hill, New York, 1976.

[5]Obermeier, F., "Berechnung aerodynamisch erzeugter Schallfelder mittels der Methode der 'Matched Asymptotic Expansions'," *Acustica*, Vol. 18, 1967, pp. 238–239.

[6]Crow, S. C., "Aerodynamic Sound Emission as a Singular Perturbation Problem," Univ. of California, UCRL 70189, Livermore,

CA, 1966.
[7]Obermeier, F., and von Schroeter, Th., "Zur Schallerzeugung durch Rotor-Wirbel-Wechselwirkungen unter Berücksichtigung der Kutta-Bedingung an der Hinterkante des Rotors," Fortschritte der Akustik, DAGA '89, pp. 595–598.
[8]Meier, G. E. A., Lent, H. M., and Löhr, K. F., "Sound Generation and Flow Interaction of Vortices with an Airfoil and a Flat Plate in Transonic Flow," *Fluid Dynamic Research*, Vol. 3, 1988, pp. 344–348.
[9]Caplot, M., and Haertig, J., "Prediction of Rotor Blade-Vortex Interaction Noise from 2D Aerodynamic Calculations and Measurements," 14th European Rotorcraft FORUM, Paper 3, Milano, Italy, Sept. 20–23, 1988.
[10]Jones, D. S., "Aerodynamic Sound Due to a Source Near a Half Plane," *Journal of the Institute of Mathematics and Its Applications*, Vol. 9, 1972, pp. 114–122.
[11]Crighton, D. G., "Radiation Properties of the Semi-Infinite Vortex Street," *Philosophical Transactions of the Royal Society of London, Series A*, Vol. 330, 1972, pp. 185–198.
[12]Howe, M. S., "Contributions to the Theory of Aerodynamic Sound, with Application to Excess Noise and the Theory of the Flute," *Journal of Fluid Mechanics*, Vol. 71, 1975, pp. 625–673.
[13]Howe, M. S., "Contributions to the Theory of Sound Production by Vortex-Airfoil Interaction, with Application to Vortices with Finite Axial Velocity Defect," *Philosophical Transactions of the Royal Society of London, Series A*, Vol. 420, 1988, pp. 157–182.
[14]Möhring, W., "On Vortex Sound at Low Mach Number," *Journal of Fluid Mechanics*, Vol. 85, 1978, pp. 685–691.
[15]Obermeier, F., "On a New Representation of Aeroacoustic Source Distribution. I. General Theory; II. Two-Dimensional Model Flows," *Acustica*, Vol. 42, 1979, pp. 56–71.
[16]Obermeier, F., "The Influence of Solid Bodies on Low Mach Number Vortex Sound," *Journal of Sound and Vibration*, Vol. 72, 1980, pp. 38–49.
[17]Powell, A., "Theory of Vortex Sound," *Journal of the Acoustical Society of America*, Vol. 36, 1964, pp. 177–194.
[18]Lin, C. C., "On the Motion of Vortices in Two Dimensions," *Applied Mathematics, Series No. 5*, Univ. of Toronto Studies, 1943.
[19]Von Friedrich-Schroeter, Th., "Untersuchung zum Einfluβ der Kutta-Bedingung auf die Schallerzeugung durch Wirbel-Körper-Wechselwirkungen," Diplomarbeit, Univ. Göttingen, 1988.
[20]Möhring, W., "Modelling Low Mach-Number Noise," *Proceedings of the IUTAM/ICA/AIAA-Symposium*, Göttingen 1979 "Mechanics of Sound Generation in Flows," Springer-Verlag, Berlin, 1979, pp. 85–95.
[21]Meier, G. E. A., and Timm, R., "Unsteady Vortex-Airfoil Interaction," AGARD-CP-386, 1985, Paper 16; see also Timm, R., "Schallentstehung bei der Wechselwirkung von Wirbeln mit einer Tragflügelunströmung," Mitteilungen aus dem Max-Planck-Institut für Strömungsforschung, Nr. 80, 1985.